Effective Medium Theory for Elastic Composites: Beyond the Quasi-Static Limit

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Recently, a great deal of attention has been attracted by phononic crystals, which are periodic composites that are acoustic and elastic analogues of photonic crystals. Some of these composites possess resonances at low frequencies. In order to describe wave propagation in these kinds of materials at frequencies near resonances, it is necessary to establish a dynamical effective medium theory (EMT). The existing EMT is only valid in the quasi-static limit and fails when resonances appear. Here, we present a dynamical EMT which simultaneously determines the elastic Lamé constants, λ and μ, and mass density, ρ, of a two-dimensional phononic crystal beyond the quasi-static limit. The origin of the band gaps of a locally resonant phononic crystal in a certain frequency range can be clearly understood. Furthermore, we can accurately predict the positions of band gaps, double-negative bands and some of the other band properties by the EMT. The validity of the theory is checked by numerical band-structure calculations for some particular crystals which have resonances in the microstructure at low frequencies.